Further Calculations of the Flutter Speed of a Fully Submerged Subcavitating Hydrofoil

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Some new calculations of the flutter speed of the Southwest Research Institute (SwRI) fully submerged subcavitating hydrofoil model are presented. Variations in lift-curve slope and in center of pressure location are found to have a most profound influence on both flutter speed and frequency. When variations in these parameters are combined with a relaxation of the Kutta condition (proposed previously), excellent agreement with the measured flutter speed is obtained.

Nomenclature

$A_{hh}, A_{h\alpha}$,
$A_{\alpha h}, A_{\alpha\alpha}$ = generalized loadings
a = distance from midchord to moment axis
a_0, a_1, a_2 = coefficients of the characteristic equation
b = semichord
$C(k)$ = Theodorsen circulation function
C_{l_α} = lift-curve slope
d = distance from moment axis to quarterchord
f_h, f_α = mode shape in heave and pitch, respectively
g = coefficient of structural damping
h = heave displacement
I_α = moment of inertia
k = reduced frequency
L_h, L_α = lift in heave and pitch, respectively
M = mass
M_h, M_α = moment in heave and pitch, respectively
r_α = radius of gyration
r_ω = frequency ratio
S_α = static moment
s = aspect ratio
V = freestream velocity
$x_{c.p.}$ = center of pressure location
x_α = distance to wing section center of gravity aft of elastic axis
y = spanwise coordinate
Z = complex frequency ratio
α = pitch displacement
β = ratio of lift-curve slopes
γ = mode shape parameter
η = magnitude of empirical factor
λ = center of pressure parameter
μ = mass coefficient
ρ = fluid density
σ_h, σ_α = finite span correction factors
ϕ = phase angle

Subscripts

α, η = quantities associated with pitching and heaving, respectively

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Introduction

THE catastrophic structural failure of the SwRI flutter model as reported by Abramson and Ransleben[1] has been the focal point of much discussion and the subject of a variety of calculations and analyses. The conditions at which catastrophic failure of this model occurred were a velocity of 48.1 knots and a frequency of 17.5 cps (corresponding to a reduced velocity of 1.48). However, in making any comparison or correlations with these data, it should be quite clearly noted that uncertainties exist even here,[2,3] so that exact agreement should be viewed with as much skepticism as would wide disagreement.

Initial attempts to calculate the flutter velocity of this model by two-dimensional and Reissner-Stevens theories failed to yield reasonable results,[1,4] although the model had an aspect ratio of five. This difficulty was not unanticipated, however, as large differences between experimental data and computed results for various flutter studies at low mass density ratios had previously been encountered and had been the subject of much controversy.[2,5] It therefore has now become rather widely accepted that, for the range of parameters pertinent to hydrofoil applications the more or less classical formulations of the unsteady hydrodynamic loads are not adequate. However, even the evidence available here is somewhat clouded since, for example, measured data on oscillatory lift and moment distributions[6,7] employed directly in a flutter analysis of the SwRI flutter model yielded a substantially overconservative prediction[1,4]; unfortunately, the accuracy of this measured data is also open to some question,[3] and the analytical techniques employed in the analysis may not have been adequate.

A somewhat different approach also was taken quite early in all of these studies by considering some relaxation of the Kutta condition[8,9]; but, again, reasonable flutter predictions were not readily forthcoming[4] and could only be obtained from this semiempirical procedure by using rather large arbitrary shifts in the phase of the circulation function.

Several years ago, Yates began the development of a modified strip-analysis method employing arbitrary spanwise distributions of lift-curve slope and center of pressure (c.p.) location.[10–14] This method can be based on the idea of employing measured data for these parameters or, alternatively, employing values calculated from lifting surface theory. Yates has applied this basic method to the SwRI flutter model,[15] obtaining a flutter velocity of approximately 38.5 knots, with a structural damping coefficient of $g = 0.02$ cps and a flutter frequency of 21.3 cps. Some further refinements and modifications appear to have yielded somewhat better agreement, but these have not yet been reported in detail.

Yet another approach has relied upon the rather full application of lifting surface theories.[16–18] The method proposed by Rowe[18] provides for some variation in satisfaction of the Kutta condition and inherently generates data on lift-curve slope and c.p. location.

It is the purpose of the present report to present yet another series of results. These are all based essentially upon strip theory, but involving various modifications to account for Kutta condition, lift-curve slope, c.p. location, etc.

Analysis

Basic Equations

For combined pitching and heaving oscillations of a hydrofoil, represented as a uniform unswept beam, the equations of motion may be put into the nondimensional form[19]

$$-\mu\gamma_h^2 h_0^* - \mu\gamma_{\alpha h} x_\alpha \alpha_0 + \mu\gamma_h^2 r_\omega^2(\omega_\alpha^2/\omega^2) \times (1+ig)h_0^* = A_{hh}^* h_0^* + A_{h\alpha}^*\alpha_0 \quad (1a)$$

$$-\mu\gamma_{\alpha h} x_\alpha h_0^* - \mu r_\alpha^2\gamma_\alpha^2\alpha_0 + \gamma_\alpha^2(\omega_\alpha^2/\omega^2) \times [1+ig]\mu r_\alpha^2\alpha_0 = A_{\alpha h}^* h_0^* + A_{\alpha\alpha}^*\alpha_0 \quad (1b)$$

where

$$\gamma_h^2 = \int_0^1 f_h^2(y^*)dy^* \quad (2a)$$

$$\gamma_\alpha^2 = \int_0^1 f_\alpha^2(y^*)dy^* \quad (2b)$$

$$\gamma_{\alpha h} = \int_0^1 f_\alpha(y^*)f_h(y^*)dy^* \quad (2c)$$

f_α, f_h are the pitching and heaving mode shapes, respectively, i.e.,

$$\alpha = \alpha_0 f_\alpha(y^*) \text{ positive nose-up} \quad (3a)$$

$$h/b = h_0^* f_h(y^*) \text{ positive downward} \quad (3b)$$

$$y^* = y/sb;\ s = \text{aspect ratio, } b = \text{semichord} \quad (4a)$$

$$\mu = M/\pi\rho b^2 \quad (4b)$$

$$r_\alpha^2 = I_\alpha/Mb^2 \quad (4c)$$

$$x_\alpha = S_\alpha/Mb \quad (4d)$$

$$r_\omega^2 = \omega_h^2/\omega_\alpha^2 \quad (4e)$$

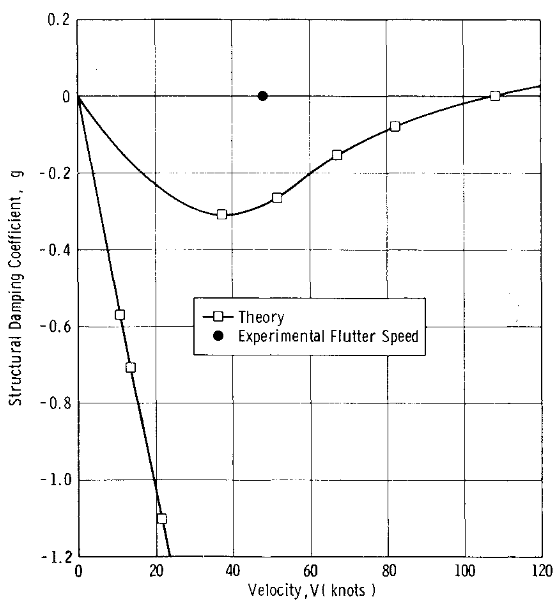

Fig. 1 *V-g* curves for case 1 (Reissner-Stevens theory).

The (nondimensional) generalized loadings are

$$A_{hh}^* = \int_0^1 L_h^* f_h^2(y^*)dy^* \quad (5a)$$

$$A_{h\alpha}^* = \int_0^1 L_\alpha^* f_\alpha(y^*)f_h(y^*)dy^* \quad (5b)$$

$$A_{\alpha h}^* = \int_0^1 M_h^* f_h(y^*)f_\alpha(y^*)dy^* \quad (5c)$$

$$A_{\alpha\alpha}^* = \int_0^1 M_\alpha^* f_\alpha^2(y^*)dy^* \quad (5d)$$

The force L^* is positive downward; the moment M^* is positive nose-up.

The determinant of the coefficients must vanish at the critical (flutter) speed; in this case,

$$a_2 z^2 + a_1 z + a_0 = 0 \quad (6)$$

where

$$z = (\omega_\alpha^2/\omega^2)(1+ig) \quad (7)$$

and

$$a_2 = \mu^2 r_\alpha^2 r_\omega^2 \gamma_\alpha^2 \gamma_h^2 \quad (8a)$$

$$a_1 = -\mu r_\omega^2 \gamma_h^2(\mu r_\alpha^2\gamma_\alpha^2 + A_{\alpha\alpha}^*) - \gamma_\alpha^2 \mu r_\alpha^2(\gamma_h^2\mu + A_{hh}^*) \quad (8b)$$

$$a_0 = (\gamma_h^2\mu + A_{hh}^*)(\mu r_\alpha^2\gamma_\alpha^2 + A_{\alpha\alpha}^*) - (\mu x_\alpha\gamma_{\alpha h} + A_{h\alpha}^*)(\mu x_\alpha\gamma_{\alpha h} + A_{\alpha h}^*) \quad (8c)$$

The standard procedure employed for determining the flutter speed from Eq. (6) is to assume a value for the reduced frequency $k = \omega b/V$ and then to calculate z and ω_α/ω. The value of g may then be found and a plot made of V vs g; the value of V at which $g = 0$ then gives the flutter velocity.

Case 1 Reissner-Stevens method[20]

Following the convention of Ref. 19, the expressions for force and moment corresponding to pitch and heave are, respectively,

$$L_h^* = \frac{L'_h/h^*}{\pi\rho\omega^2 b^3} = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik[C(k) + \sigma_h]\right\} = \bar{L}_h^* e^{i\alpha_{Lh}} \quad (9a)$$

$$L_\alpha^* = \frac{L'_\alpha/\alpha}{\pi\rho\omega^2 b^3} = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right][C(k) + \sigma_\alpha]\right\} = \bar{L}_\alpha^* e^{i\alpha_{L\alpha}} \quad (9b)$$

$$M_h^* = \frac{M'_h/h^*}{\pi\rho\omega^2 b^4} = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - \left(a + \frac{1}{2}\right) \times ik[C(k) + \sigma_h]\right\} = \bar{M}_h^* e^{i\alpha_{Mh}} \quad (9c)$$

$$M_\alpha^* = \frac{M'_\alpha/\alpha}{\pi\rho\omega^2 b^4} = -\frac{2}{k^2}\left\{\frac{1}{2}ik\left(\frac{1}{2} - a\right) - \frac{1}{2}k^2\left(\frac{1}{8} + a^2\right) - \left(a + \frac{1}{2}\right)\cdot\left[1 + ik\left(\frac{1}{2} - a\right)\right] \times [C(k) + \sigma_\alpha]\right\} = \bar{M}_\alpha^* e^{i\alpha_{M\alpha}} \quad (9d)$$

The procedure required to calculate the finite span correction factors σ_h and σ_α is given in Ref. 20 (for two-dimensional theory, $\sigma_h = \sigma_\alpha = 0$).

The lift and moment calculated for the SwRI flutter model are found to be in agreement with those obtained previously with a different computer program.[1] The V-g curve is given in Fig. 1; the flutter speed lies between two previously calculated values, all of which are based upon

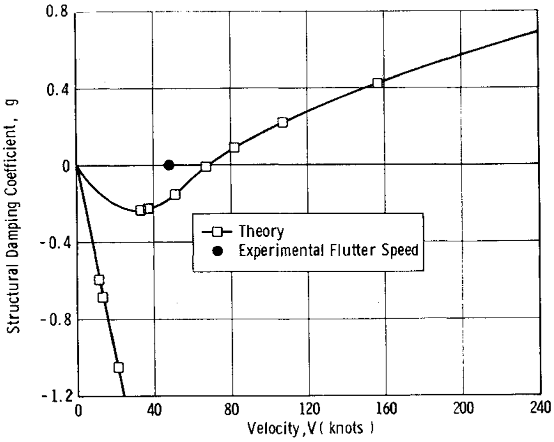

Fig. 2 V-g curves for case 2.

certain data read from curves given in Ref. 20, so that the basic computer program to be employed in the calculation of the curves to follow is confirmed to be correct. The measured mode shapes and characteristics of the SwRI flutter model, as employed in these calculations, are given in Tables 1a and 1b.

Case 2 Calculation using experimental data for steady-state c.p. location

The expressions for lift and moment are now written as

$$L_h^* = -(2/k^2)\{-k^2/2 + ik[C(k) + \sigma_h]\} \quad (10a)$$

$$L_\alpha^* = -(2/k^2)\{\tfrac{1}{2}(ik + k^2a) + [1 + ik(\tfrac{1}{2} - a)] \times [C(k) + \sigma_\alpha]\} \quad (10b)$$

$$M_h^* = -(2/k^2)\{ak^2/2 - (a - x_{c\cdot p\cdot})ik[C(k) + \sigma_h]\} \quad (10c)$$

$$M_\alpha^* = -(2/k^2)\{\tfrac{1}{2}ik(\tfrac{1}{2} - a) - \tfrac{1}{2}(k^2 + a^2) - (a - x_{c\cdot p\cdot})[1 + ik(\tfrac{1}{2} - a)]\cdot[C(k) + \sigma_\alpha]\} \quad (10d)$$

where $x_{c.p.}$ is the location of the center of pressure, dependent upon the spanwise positions, or obtained from the experimental data of Ref. 6. In the moment expressions, Eqs. (10c) and (10d), the term $a - x_{c.p.}$ has been used in place of the term $\frac{1}{2} + a$ as the lift force is to be taken as acting at $x_{c.p.}$ rather than at the quarter-chord point $(-\frac{1}{2}b)$. The V-g curve is given in Fig. 2, indicating a flutter speed of approximately 67 knots (compared with the experimental value of 48 knots).

Case 3 Calculation using experimental data for oscillating c.p. location

The forces will be considered to be the same as those of the classical theory of Case 1, but the moments will be taken as

$$M_h^* = \bar{M}_h^* e^{i\alpha_{Mh}} - \bar{M}_h^* \cos(\alpha_{Mh} - \alpha_{Lh})e^{i\alpha_{Lh}} + x_{c.p.h}^* L_h^* e^{i\alpha_{Lh}} \quad (11a)‡$$

$$M_\alpha^* = \bar{M}_\alpha^* e^{i\alpha_{M\alpha}} - \bar{M}_\alpha^* \cos(\alpha_{M\alpha} - \alpha_{L\alpha})e^{i\alpha_{Lh}} + x_{c.p.\alpha}^* L_\alpha^* e^{i\alpha_{Lh}} \quad (11b)$$

The unsteady c.p. locations are taken as $x_{c.p.h}^*$, $x_{c.p.\alpha}^*$ in these expressions and defined[6] so that

$$L_\alpha^* x_{c.p.\alpha}^* = \bar{M}_\alpha^* \cos(\alpha_{Mh} - \alpha_{Lh}) \quad (12a)$$

$$L_h^* x_{c.p.h}^* = \bar{M}_h^* \cos(\alpha_M - \alpha_{L\alpha}) \quad (12b)$$

‡ M_h^*, M_α^*, α_{Mh}, $\alpha_{M\alpha}$, L_h^*, L_α^*, α_{Lh}, $\alpha_{L\alpha}$ are given by Eqs. (9a–9d) with $\sigma_h = \sigma\alpha = 0$.

Table 1a SwRI flutter model-measured mode shapes (in air)

	y/L	f_h	f_α
Root	0	0	0
	0.1	0.025	0.213
	0.2	0.075	0.403
	0.3	0.158	0.570
	0.4	0.260	0.710
	0.5	0.370	0.825
	0.6	0.488	0.915
	0.7	0.610	0.966
	0.8	0.738	0.990
	0.9	0.870	0.999
Tip	1.0	1.000	1.000

where

$$x_{c.p.h}^* = 2\bar{x}_{c.ph} - \tfrac{1}{2} - d \quad (13a)$$

$$x_{c.p.\alpha}^* = 2\bar{x}_{c.p.\alpha} - \tfrac{1}{2} - d \quad (13b)$$

and $\bar{x}_{c.p.h}$, $\bar{x}_{c.p.\alpha}$ are the oscillatory center of pressure locations corresponding to heave and pitch motions, respectively, in fractions of chord measured from the leading edge; d is the distance between the moment axis and the quarter-chord point. For the SwRI flutter model

$$\tfrac{1}{2} + d = 0.31 \quad (14)$$

Values of $x_{c.p.h}^*$, $x_{c.p.\alpha}^*$ are given in Table 2. The V-g curve is given in Fig. 3, indicating a flutter speed of approximately 70 knots.

Table 1b SwRI flutter model characteristics[1]

Model parameter	Measured value
Aspect ratio	5.00
Semichord	0.50 ft
Elastic axis location a	−0.50
Center of gravity location x_α (from aeroelastic axis)	0.524
Radius of gyration r_α^2	0.512
Bending stiffness[a] EI	3.40×10^6 lb-in^2
Torsional stiffness[a] GJ	0.973×10^6 lb-in^2
Frequency ratio ω_h/ω_α	0.490
Torsional frequency ω_α	20.5 cps
Total weight (wing only)	121.2 lb

[a] The calculated ω_α and ω_h are about 18.7 and 11.2 cps, respectively. The calculated ω_h/ω_α is about 0.60. However, the measured frequencies are possibly more accurate than the calculated values based on measured EI and GJ.

Case 4 Calculation using modified values of lift-curve slope and c.p.

It is well known, in steady flow, that the actual lift-curve slope of a foil is somewhat less than the theoretical value of 2π, thus having the effect of reducing the value of the circulation. Denoting the ratio of actual to theoretical lift-curve slopes by β, the Theodorsen function may be multiplied by this ratio. (No additional corrections are to be applied to the Reissner-Stevens three-dimensional factor.) Then, con-

Table 2 Values of oscillatory c.p. locations in semichords[6]

$1/k$	$\bar{k}$	$2\bar{x}_{c.p.h}$	$2\bar{x}_{c.p.\alpha}$	$x_{c.p.h}^*$	$x_{c.p.\alpha}^*$
0.4	2.5	0.85	0.92	0.54	0.61
0.8	1.25	0.630	0.76	0.32	0.45
1.0	1.00	0.560	0.70	0.25	0.39
1.5	0.667	0.480	0.58	0.17	0.27
1.8	0.556	0.48	0.53	0.17	0.22
2.0	0.5	0.48	0.50	0.17	0.19

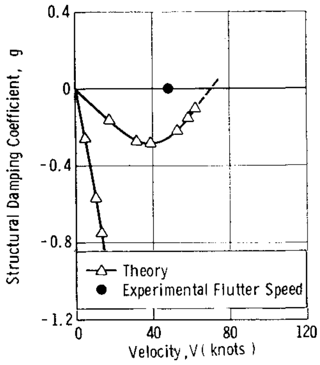

Fig. 3 ***V-g* curves for case 3.**

sidering the lift force to act at the center of pressure, the expressions for lift and moment become

$$L_h^* = -(2/k^2)\{-k^2/2 + ik[\beta C(k) + \sigma_h]\} \quad (15a)$$

$$L_\alpha^* = -(2/k^2)\{\tfrac{1}{2}(ik + k^2a) + [1 + ik(\tfrac{1}{2} - a)][\beta C(k) + \sigma_\alpha]\} \quad (15b)$$

$$M_h^* = -(2/k^2)\{ak^2/2 - (a - x_{c.p.})ik[\beta C(k) + \sigma_h]\} \quad (15c)$$

$$M_\alpha^* = -(2/k^2)\{\tfrac{1}{2}ik(\tfrac{1}{2} - a) - \tfrac{1}{2}k^2(\tfrac{1}{8} + a^2) - (a - x_{c.p.})[1 + ik(\tfrac{1}{2} - a)]\cdot[\beta C(k) + \sigma_\alpha]\} \quad (15d)$$

Various values of $C_{l\alpha}$ and $x_{c.p.}$ are given in Tables 3a and 3b.

Table 3a Experimental values for steady-state lift-curve slope $C_{l\alpha}$ and center of pressure, $x_{c.p.}$ (Ref. 6)

y^*	$C_{l\alpha}$, rad	$x_{c.p.}$, semichords
0	4.01	−0.522
0.1	4.01	−0.522
0.2	4.01	−0.536
0.3	4.01	−0.540
0.4	4.01	−0.544
0.5	4.01	−0.552
0.6	4.01	−0.564
0.7	3.724	−0.580
0.8	3.438	−0.596
0.9	2.58	−0.632
1.0	0	−0.640

The data in Table 3a are derived from experiments[6] while those in Table 3b are derived from lifting surface theory.[15]

V-g curves calculated by using these two sets of $C_{l\alpha}$ and $x_{c.p.}$ values are given in Figs. 4a and 4b. The first, employing measured foil data, gives a flutter speed of approximately 100 knots, whereas the second, employing calculated foil data, gives a flutter velocity of approximately 76 knots. Comparing these results directly with those of case 2 shows that the effect of modifying the lift-curve slope in this manner is to raise the flutter velocity.

Table 3b Theoretical values for steady-state lift-curve slope $C_{l\alpha}$ and center of pressure $x_{c.p.}$ for the SwRI flutter model

y^*	$C_{l\alpha}$, rad	$x_{c.p.}$, semichords
0	4.85	−0.531
0.1	4.84	−0.531
0.2	4.80	−0.533
0.3	4.71	−0.535
0.4	4.60	−0.536
0.5	4.44	−0.542
0.6	4.21	−0.550
0.7	3.86	−0.563
0.8	3.36	−0.578
0.9	2.54	−0.605
1.0	0	−0.646

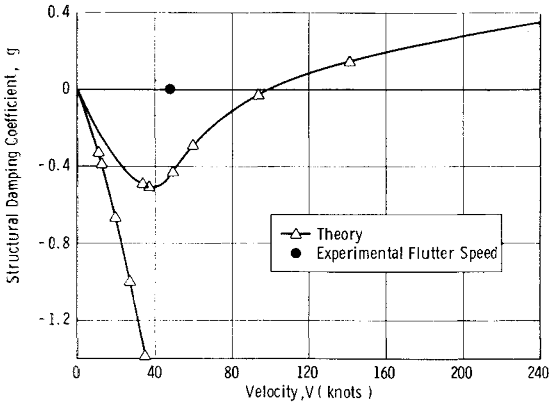

Fig. 4a ***V-g* curves for case 4a.**

***Case 5 Calculation using Yates' modified strip theory*[10]**

According to Yates,[10] the constant factor $\frac{1}{2} - a$ in the downwash terms of a conventional strip theory analysis could be replaced by

$$\lambda = \beta + x_{c.p.} - a \quad (16)$$

which varies along the span of the foil. Since this modification includes three-dimensional effects, σ_α and σ_h are to be taken as zero. The expressions for lift and moment then become

$$L_h^* = -(2/k^2)\{-k^2/2 + ikC(k)\} \quad (17a)$$

$$L_\alpha^* = -(2/k^2)\{\tfrac{1}{2}(ik + k^2a) + (1 + ik\lambda)C(k)\} \quad (17b)$$

$$M_h^* = -(2/k^2)\{ak^2/2 - (\tfrac{1}{2} + a)ikC(k)\} \quad (17c)$$

$$M_\alpha^* = -(2/k^2)\{\tfrac{1}{2}ik\lambda - \tfrac{1}{2}k^2(\tfrac{1}{8} + a^2) - (a + \tfrac{1}{2})(1 + ik\lambda)C(k)\} \quad (17d)$$

Two subcases are now considered. Case 5a is based on the use of the values derived by Yates[15] and given in Table 3b. The *V-g* curve corresponding to this calculation is shown in Fig. 5a and gives a flutter velocity of approximately 30 knots with $g \cong 0$ and 32 knots with $g = 0.02$; this compares with a value of 38.5 knots with $g = 0.02$ as computed by Yates himself.[15] Note, however, that the slope of the critical *V-g* curve for small values of g is so flat that only very slight changes in the computed points would be necessary to alter the flutter velocity by a very large amount. Further, Yates employed six calculated modes (two bending and four torsion) in his analysis, whereas the present calculations are based on two measured mode shapes (one bending and one

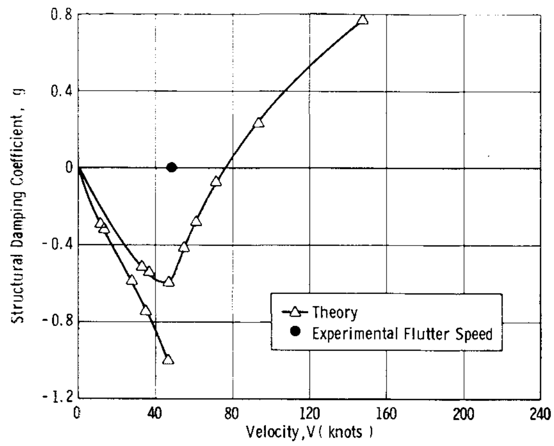

Fig. 4b ***V-g* curves for case 4b.**

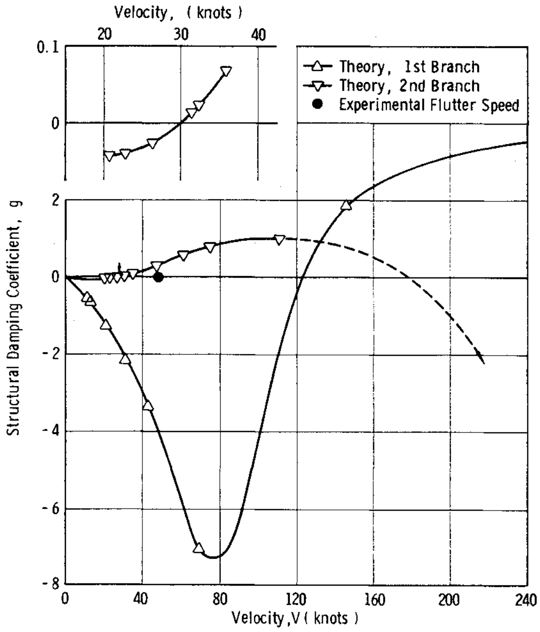

Fig. 5a ***V-g* curves for case 5a.**

torsion).§ In any event, it would appear that the modified strip theory proposed by Yates (and as employed in this report) is overconservative by a significant degree.

Case 5b employs the measured steady state values shown in Table 3a. The corresponding *V-g* curve is shown in Fig. 5b, giving a flutter speed of approximately 26 knots.

Case 6 Calculations based on application of the generalized Kutta condition

As proposed a number of years ago, [4,8,9,21] a generalization of the Kutta condition can be formulated in terms of a factor λ_1 applied to the trailing-edge tangential velocity. Defining $\lambda_1 = 1 - \eta e^{i\phi_0}$, and replacing the constant factor $\frac{1}{2} + a$ by $a - x_{c.p.}$ as was done in case 2, the lift and moment become

$$L_h^* = -(2/k^2)\{-k^2/2 + ik\eta e^{i\phi_0}[C(k) + \sigma_h]\} \quad (18a)$$

$$L_\alpha^* = -(2/k^2)\{\tfrac{1}{2}(ik + k^2 a) + [1 + ik(\tfrac{1}{2} - a)]\eta e^{i\phi_0}[C(k) + \sigma_\alpha]\} \quad (18b)$$

$$M_h^* = -(2/k^2)\{ak^2/2 - (a - x_{c.p.})ik\eta e^{i\phi_0}[C(k) + \sigma_h] - i\lambda_1 k/2\} \quad (18c)$$

$$M_\alpha^* = -(2/k^2)\{\tfrac{1}{2}ik(\tfrac{1}{2} - a)\eta e^{i\phi_0} - (\lambda_1/2) - \tfrac{1}{2}k^2(\tfrac{1}{8} + a^2) - (a - x_{c.p.}) \times [1 + ik(\tfrac{1}{2} - a)]\eta e^{i\phi_0}[C(k) + \sigma_\alpha]\} \quad (18d)$$

We shall assume that

$$\eta = \beta/\cos\phi_0 \quad (19a)$$

$$\beta = C_{l\alpha}/2\pi \quad (19b)$$

and shall select specific values of ϕ_0.

Case 6a is based on the use of measured data for $C_{l\alpha}$ and $x_{c.p.}$ (Table 3a), with *V-g* curves corresponding to several values of ϕ_0 shown in Fig. 6a. The value of $\phi_0 = 0$ gives a flutter speed of approximately 42 knots, compared with the experimental value of 48.1 knots; a calculation using the value of $\phi_0 = 10°$ gives almost exactly 48 knots. It is interesting to note that the Kutta condition modification alone (e.g., without any simultaneous correction on $C_{l\alpha}$ and $x_{c.p.}$) gives computed results that agree with the actual hydrofoil flutter velocity of 48 knots only for the somewhat unrealistic value of $\phi_0 = -30°$.[4] Even for $\phi_0 = 0$, however, these new results are quite good, especially if one considers that we have spoken only of the flutter speed corresponding to $g = 0$; in fact, based on zero forward speed tests, the damping of the SwRI flutter model was estimated to be $g_h = 0.016$ and $g_\alpha = 0.042$ in air, and is roughly estimated to be $g_h = 0.070$ and

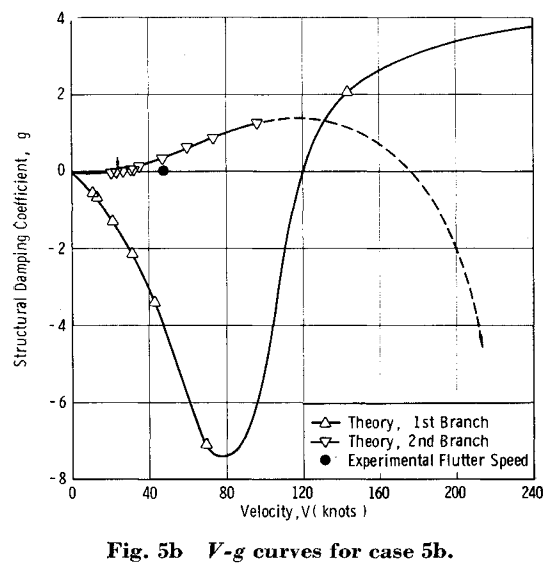

Fig. 5b ***V-g* curves for case 5b.**

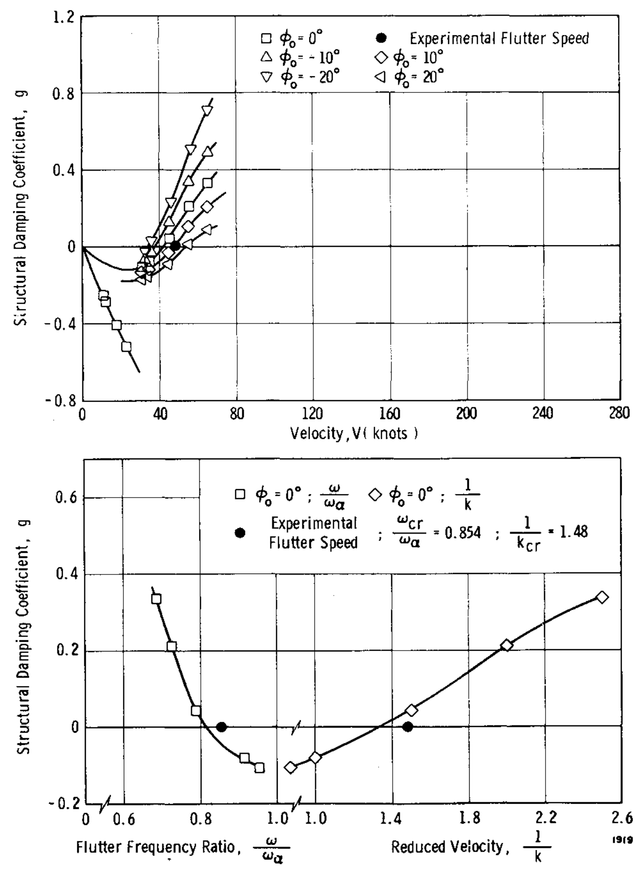

Fig. 6a ***V-g* curves and $\omega_{cr}/\omega_\alpha$ and $1/k_{cr}$ curve for case 6a.**

§ Some rather simple considerations show that our use of the two measured modes should give reasonably accurate results. Any discrepancies arising between using calculated and measured modes are the result of differences in the dominant terms rather than an insufficient number of modes.

Table 4 Summary of results

Case no.	Brief description	Flutter speed V_f, knots	Critical frequency ω/ω_α
Experiment	SwRI flutter model ($b = 0.5$ ft, $\omega_\alpha = 20.5$ cps)	48.1	0.854
1	Reissner-Stevens theory	108	...
2	Experimental (steady) $x_{c.p.}$, $C_{l_\alpha} = 2\pi$; $\sigma_j \cdot j = \alpha, h$	67	...
3	Unsteady $x_{c.p.\alpha}{}^*$, $x_{c.p.h}{}^*$	70	...
4a	Exp. (steady) $x_{c.p.}$, $C_{l\alpha}$; $\sigma_j = 0$	100	...
4b	Exp. (steady) $x_{c.p.}$, C_{lj}; σ_j	76	...
5a	Yates theory; Yates' $x_{c.p.}$, $C_{l\alpha}$	30	...
5b	Yates theory; exp. (steady) $x_{c.p.}$, $C_{l\alpha}$	26	...
6a	Exp. (steady) $x_{c.p.}$, $C_{l\alpha}$; generalized Kutta condition, $\phi_0 = 0$	42	0.82
6b	Yates $x_{c.p.}$, $C_{l\alpha}$; generalized Kutta condition, $\phi_0 = 0$	45	0.82
6c	Exp. (steady) $C_{l\alpha}$; $x_{c.p.} = -\frac{1}{2}$ generalized Kutta condition, $\phi_0 = 0$	45	0.82

$g_\alpha = 0.047$ in water.¶ Calculated data corresponding to the critical frequency are also shown in Fig. 6a, giving a value of $\omega/\omega_\alpha = 0.82$ and $1/k = 1.34$ (for $g = 0$), which are in good agreement with the measured values.

Case 6b is based on the use of Yates' calculated data for $C_{l\alpha}$ and $x_{c.p.}$ (Table 3b), with V-g curves corresponding to several values of ϕ_0 shown in Fig. 6b, along with data for critical frequency. The value of $\phi_0 = 0$ (for $g = 0$) gives a flutter velocity of 45 knots and $\omega/\omega_\alpha = 0.82(1/k = 1.43)$. These are again in very good agreement with the measured values.

Finally, case 6c differs from case 6a only in that the center of pressure is taken as $x_{c.p.} = -\frac{1}{2}$, with the results being shown in Fig. 6c. Again, with $\phi_0 = 0$ and $g = 0$, the flutter speed is 45 knots and $\omega/\omega_\alpha = 0.82$ $(1/k = 1.43)$.

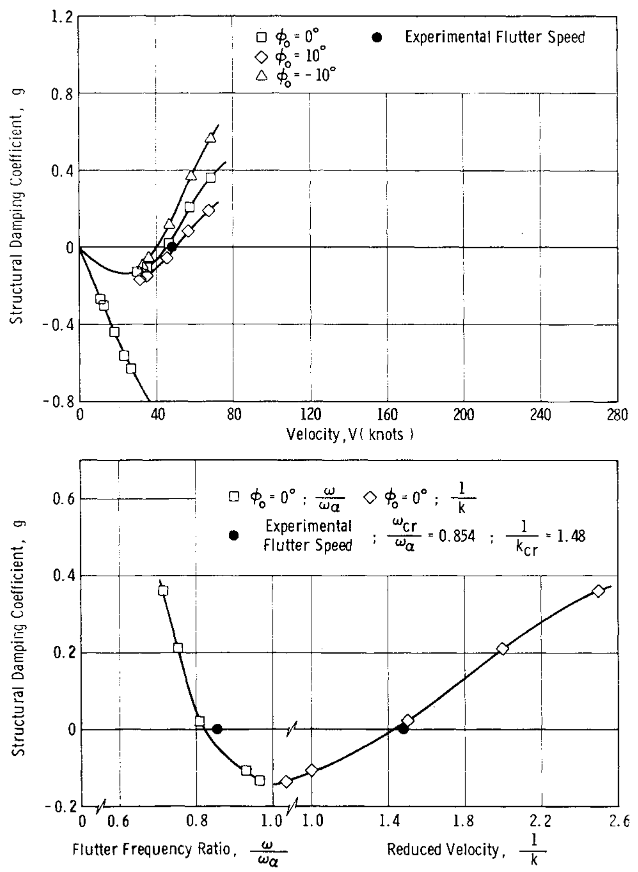

Fig. 6b V-g curves and $\omega_{cr}/\omega_\alpha$ and $1/k_{rc}$ curve for case 6b.

¶ Data provided by G. E. Ransleben Jr.

Discussion and Conclusions

A summary of the results obtained from the various cases studied is given in Table 4. Of these, only case 6 gives results that are closely comparable with the measured values of flutter speed and frequency.

There can be no question that variations in lift-curve slope and in center of pressure location, as suggested by Yates,[15] have a most profound influence on the flutter speed and frequency of a foil having parameters similar to those of the SwRI flutter model.[1] Although the calculated results presented in this report are not in exceptionally close agreement with those of Yates, it appears this may be the consequence, in part, of sensitivity of the computations and also of the use of measured modes in the present calculations and of calculated modes by Yates. Both sets of results, however, lead one to conservative predictions, and this in itself is a

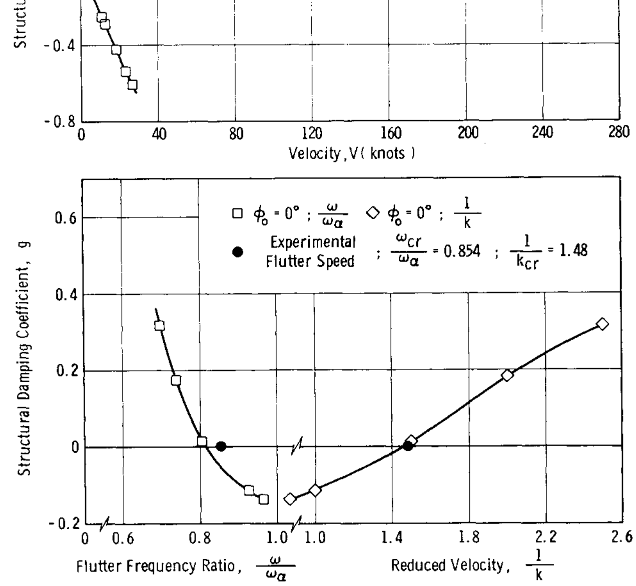

Fig. 6c V-g curves and $\omega_{cr}/\omega_\alpha$ and $1/k_{rc}$ curve for case 6c.

significant advance over practically all previous results. The difference between using the measured values of lift-curve slope and the c.p. location and that of using those calculated from lifting surface theory is apparently not large. Yates' method certainly deserves further study and application.

The semiempirical method based on generalization of the Kutta conditions,[8,9,21] and used here in conjunction with lift-curve slope and c.p. modifications, appears to give excellent results (case 6). As with the Yates' method, the differences arising from various $C_{l\alpha}$ and $x_{c.p.}$ values are small. An essential point is again that the flutter predictions are *conservative*, as well as being in close agreement with the measured flutter speed and frequency. It would mean that the applications of the procedures employed in case 6 to other hydrofoil flutter studies would be quite useful and revealing, if done prudently.

The role of structural damping could be significant; in case 6, the inclusion of small positive damping would serve to improve even further the agreement between calculated and measured flutter speed.

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